Multi-Resolution Spatial Methods on Spheres

HAO-YUN HUANG
Institute of Statistics, National Tsing Hua University,
Hsin-Cheng Huang
Institute of Statistical Science, Academia Sinica,
and
CHING-KANG ING
Institute of Statistics, National Tsing Hua University,

Abstract

We consider spatial modeling for massive global data observed at the surface of the earth. We develop a new basis in the thin-plate spline (TPS) function space on the sphere. Our basis functions are ordered in terms of their degrees of smoothness, which provide a multi-resolution representation with the resolution controlled by the number of basis functions. By applying a small to a moderately large number of basis functions, we not only gain statistical efficiency by dimension reduction, but we can also reduce the computational complexity enormously over the TPS method. When the underlying function lies in a Sobolev space on the sphere, we show that the optimal number of basis functions is much smaller than the sample size. In fact, we achieve the same convergence rate as the TPS method. We further introduce a spatial spherical process with a stationary spherical Gaussian process added, resulting in a spatial mixed-effects model on the sphere. The conditional Akaike information criterion is applied to select the number of basis functions and to determine whether the spatial Gaussian process is needed. Global data of Sea Surface Temperature observed from a satellite is used for illustration.