On Some Resampling Procedures with the Empirical Beta Copula

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Abstract

The empirical beta copula, introduced in Segers et al. [2], can be considered as a simple but effective way of correcting and smoothing the empirical copula. No smoothing parameter needs to be chosen. The empirical beta copula turns out to be a special case of the empirical Bernstein copula, but in contrast to the latter, it is always a genuine copula. Moreover, it is extremely simple to simulate samples from it. It has been shown in [2] that the asymptotic distribution of the empirical beta copula is the same as that of the empirical copula, but in small samples, it performs better both in terms of bias and variance. Moreover, there seems little to be gained from using Bernstein smoothers at other polynomial degrees m, except in special cases such as the independence copula.

Because of the above nice properties, it is reasonable to expect that our smoothing procedure might also have a beneficial effect on the accuracy of resampling schemes for the empirical copula process (Bücher and Dette [1]). More specifically, testing procedures based on the empirical copula typically rely on the bootstrap for the computation of the critical values of the test statistic. For finite samples, the accuracy is often not very good: the actual size of the test may differ greatly from the nominal one.

In this paper, we first show the asymptotic equivalence of several related bootstrapped processes. Then we investigate the accuracy of the bootstrap resampling schemes based on the empirical beta copulas with small sample sizes; more specifically, we consider confidence intervals for some functionals such as rank correlation coefficients and dependence parameters of several well-known copula families, and goodness-of-fit testing problems including testing exchangeability. And we compare the performance of three methods: standard asymptotic approximation, standard bootstrapping and resampling from the empirical beta copula.

References

[1] A. Bücher and H. Dette. A note on bootstrap approximations for the empirical copula process. *Statistics and Probability Letters*, Vol. 80, pp. 1925–1932, 2010.

[2] J. Segers, M. Sibuya, and H. Tsukahara. The empirical beta copula. *Journal of Multivariate Analysis*, Vol. 155, pp. 35–51, 2017.