

Möbius Transformation and Its Use for Generating Skew Directional Distributions

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Abstract

Directional Statistics deals with

- (1) angular observations such as wind direction and the orientation of migrating birds,
- (2) bivariate angular observations such as wind directions measured at two locations simultaneously or at a location at two points in time,
- (3) combination of angular and linear observations such as wind direction and speed,
- (4) observations on the disc such as wind direction and ozone concentration, i.e. data which include angular observations.

In the talk we pick up the following topics: Skew or asymmetric distributions on the circle can be generated by applying a Möbius transformation from circle to itself to symmetric circular distributions. For example Kato and Jones (2010) used this method for the von Mises distribution. Jones (2004) applied the inverse Möbius transformation from disc to itself to the bivariate spherically symmetric beta distribution to get a skew distribution on the disc. Uesu and Shimizu (2010) considered a multivariate generalization of univariate and bivariate work to the case of hyper-disc using a conformal mapping from hyper-disc to itself.