On the Minimum Entropy of a Mixture of Unimodal and Symmetric Distributions

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Abstract

Progressive encoding of a signal generally involves an estimation step, designed to reduce the entropy of the residual of an observation over the entropy of the observation itself. Oftentimes the conditional distributions of an observation, given already-encoded observations, are well fit within a class of symmetric and unimodal distributions (e.g. the two-sided geometric distributions in images of natural scenes, or symmetric Paretian distributions in models of financial data). It is common practice to choose an estimator that centers, or aligns, the modes of the conditional distributions, since it is common sense that this will minimize the entropy, and hence the coding cost of the residuals. But with the exception of a special case, there has been no rigorous proof. In this talk, I will prove that the entropy of an arbitrary mixture of symmetric and unimodal distributions is minimized by aligning the modes. The result generalizes to unimodal and rotation-invariant distributions in R^n. I will also illustrate the result through some experiments with natural images.