On A Coin-Tossing Problem

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Abstract

Let $X, X_1, X_2, \cdots$ be i.i.d. $P(X=1)=P(X=-1)=1/2$, $S_n=\sum_{j=1}^{n} X_j$.

By CLT, $\frac{S_n}{\sqrt{n}} \xrightarrow{dist} N(0,1)$. Hence $\left|\frac{S_n}{\sqrt{n}}\right| \xrightarrow{dist} |N(0,1)|$.

Since $E\left|\frac{S_n}{\sqrt{n}}\right|^2=1$, $n=1,2,\cdots$,

$$E\left|\frac{S_n}{\sqrt{n}}\right| \to E\left|N(0,1)\right| = \frac{2}{\sqrt{2\pi}}\int_0^\infty t\cdot e^{-t^2/2}dt=\sqrt{\frac{2}{\pi}}.$$

But how fast? We will prove that

$$\sqrt{\frac{2}{\pi}}-E\left|\frac{S_{2n}}{\sqrt{2n}}\right| \le \sqrt{\frac{2}{\pi}}\cdot\frac{1}{8n}.$$