Selecting the Last Consecutive Records in a Record Process

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June, 2008

Abstract

Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables with a continuous function F as their common distribution function. Observing $X_1, X_2, \ldots$ sequentially, we say a record occurs at time n if $X_n > \max_{1\le i\le n-1} X_i$. Set $I_1 = 1$ and $I_n = I_{\{X_n > \max_{1\le i\le n-1} X_i\}}$ for $n > 1$. Then $I_n = 1$ if a record occurs at time n, and $I_n = 0$, otherwise. For the sequence $I_1, I_2, \ldots$, it is well known that they are independent Bernoulli random variables with $E(I_n) = 1/n$. Moreover, we say a consecutive records occurs at time n if $I_{n-1}I_n = 1$. Let $S_n = I_1I_2 + I_2I_3 + \cdots + I_{n-1}I_n$. We can prove that $S_n \xrightarrow{a.s.} S$, where $S \sim Pio(1)$. More generally, if $I_1, I_2, \ldots$ is a sequence of i.i.d. Bernoulli random variables with $E(I_n) = \lambda/(\lambda + n - 1)$, $n = 1, 2, \ldots$ $(\lambda > 0)$, then $S_n = I_1I_2 + I_2I_3 + \cdots + I_{n-1}I_n \xrightarrow{a.s.} S$ as $n \to \infty$, where $S \sim Poi(\lambda)$. Inspired by the above result, in this talk we want to study the following problem: Find an optimal strategy to maximize the probability of selecting the last consecutive records in $I_1, I_2, \ldots$, i.e., the last n with $I_{n-1}I_n = 1$.