Bayesian Inference of Variance Components in Generalized Linear Mixed Models

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Abstract

Longitudinal and correlated data are commonly modeled with generalized linear mixed models (GLMM) which contain both fixed and random effects. The source of the random effect may be the genetic heredity, familial aggregation, or individual heterogeneity. The inference of its variance component is usually difficult due to the dimension of its covariance matrix (more than one random effect) and the complexity of the likelihood function. In this talk, I will discuss briefly the restricted maximum likelihood (REML) estimation of variance components, and focus mainly on the Bayesian approach with posterior distribution. I will first demonstrate the specification of prior distribution on the random covariance matrix. Previous literature has offered two reference priors, the approximate uniform shrinkage prior and approximate Jeffreys' prior. Both are formulated based on the approximated likelihood function. I will then show, under generalized linear mixed models, the posterior mode under Jeffrey's prior is asymptotically closer to REML estimate than the mode under uniform shrinkage prior does. In fact, the relative distance converges to a positive integer for any square random matrix. In other words, the posterior mode under Jeffreys' prior and REML estimate are asymptotically equivalent. The formal Bayesian inference of the variance components can be conducted using posterior samples obtained by Markov chain Monte Carlo method. Finally, we consider a real application and simulation studies for the purpose of illustration.

Key words: GLMM; Jeffreys' prior; reference prior; REML; uniform shrinkage prior; variance components