SIIE: Sliced Inverse Interval Estimation for Sufficient Dimension Reduction

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Abstract

Standard slice-based Sufficient Dimension Reduction (SDR) methods typically reduce response-defined slices to selected moment summaries, potentially omitting distributional features not encoded by those moments. To address this limitation, we propose Sliced Inverse Interval Estimation (SIIE), a framework inspired by Symbolic Data Analysis (SDA). Instead of compressing slices into points, SIIE represents each slice by a principal-axis-oriented symbolic box using within-slice principal directions and directional extrema or interquantile ranges. We construct a new centred symbolic kernel that combines slice location, orientation, and directional spread. We develop Min-Max (SIIE-MM) and quantile (SIIE-Q) versions; the quantile formulation provides range-level robustness by reducing the direct influence of extreme projected observations on directional widths. Under explicit sufficient conditions, its column space is contained in the central subspace, with full recovery under rank-d coverage. We further establish root-n consistency of the quantile kernel estimator for fixed slice boundaries and affine equivariance of the sample estimator on its identifiable domain. An exact symmetric Gaussian example shows that SIIE-Q recovers a direction missed by sliced inverse regression (SIR). Extensive simulations and five real data analyses demonstrate that SIIE is highly competitive with existing inverse moment-based and related SDR methods in several mechanism-specific settings, particularly under fixed-predictor contamination and directional range or endpoint structure. SIIE-Q also achieves the lowest cross-validated normalized prediction error in three of the five applications. These findings support SIIE as a competitive, complementary spread-oriented extension of inverse regression.

Keywords:
Central subspace, Interval-valued data, Sliced inverse regression, Symbolic data analysis