

國立高雄大學 104 學年度研究所碩士班招生考試試題

科目：微積分
考試時間：100 分鐘

系所：統計學研究所
本科原始成績：100 分

是否使用計算機：否

1. Evaluate the following limits:

(a) (10%) $\lim_{x \rightarrow \infty} (1 - \frac{a}{x})^x$

(b) (10%) $\lim_{x \rightarrow \infty} (\frac{1}{x} \ln \frac{1}{x})$

2. (10%) Find the second derivative of the function $f(x) = \sqrt{x} \tan \sqrt{x}$.

3. (10%) Find the 5th degree Taylor polynomials centered at 0 for the function $f(x) = \sqrt{1+x^2}$.

4. Evaluate the following integrals:

(a) (10%) $\int \tan^2 x dx$

(b) (10%) $\int x \ln x dx$

5. (15%) Use double integration to calculate the area of the region Ω enclosed $y = x^2$ and $x + y = 2$.

6. (10%) Determine whether $\sum_{k=0}^{\infty} \frac{2^k + k^4}{3^k}$ convergence or divergence.

7. (15%) Find the area of the largest rectangle that can be inscribed in a circle of radius 5.