科目：微積分　　　　　系所：統計學研究所(無組別)　　　是否使用計算機：否

考試時間：100 分鐘　　　本科原始成績：100 分

每題 10 分，共 10 題。

1. $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}} = ?$

2. Determine the open intervals of the x-axis on which the function
$$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$$
is increasing.

3. Show that
$$\int \sin^3 x \ dx = \frac{1}{3}\cos^3 x - \cos x + C$$

4. Find the area of the region bounded by the line $y = x/2$ and by the parabola $y^2 = 8 - x$.

5. Evaluate the integral
$$\int_0^\infty x^{20} e^{-3x} \ dx$$

6. Suppose that $F(x)$ is an antiderivative of $(\sin x)/x, \ x > 0$. Express
$$\int_1^3 \frac{\sin(2x)}{x} dx$$
In terms of F.

7. Show that
$$\int_{-\infty}^\infty \exp\left(\frac{-x^2}{2}\right) dx = \sqrt{2\pi}$$

8. Determine
$$\sum_{n=1}^\infty \frac{(2n)!}{n!\,n!}$$

9. Find the Taylor series of $f(x) = e^x$ at $x = 1$.

10. Find the greatest and smallest values that the function $f(x,y) = xy$ takes on the ellipse
$$\frac{x^2}{8} + \frac{y^2}{2} = 1.$$