

國立高雄大學 101 學年度研究所碩士班招生考試試題

科目：基礎數學
考試時間：100 分鐘

系所：
統計學研究所(統計組)
本科原始成績：100 分

是否使用計算機：否

In the following, I_p denotes the identity matrix of order p and A' denotes the transpose of a matrix A .

1. (20%) Evaluate the following limits:

$$(a) \lim_{x \rightarrow 0^+} (\sin x)^x \quad (b) \lim_{x \rightarrow 0^+} \frac{\exp(-1/x)}{x}$$

2. (15%) Apply the method of Lagrange multipliers to find the maximum value of

$$f(x_1, x_2, x_3) = (x_1 x_2 x_3)^2 \quad \text{subject to} \quad x_1^2 + x_2^2 + x_3^2 = c^2,$$

where c is a constant.

3. (20%) Evaluate the following integrals:

$$(a) \int_{-\infty}^{\infty} \frac{e^{-x}}{(1+e^{-x})^2} dx \quad (b) \int_0^1 \frac{1}{\sqrt{1+x^2}} dx$$

4. (10%) Find the inverse of

$$H = I_p - \frac{2uu'}{u'u},$$

where u is a $p \times 1$ nonzero vector.

5. (10%) Show that $H = I_p - \frac{1}{p}J_p$ is idempotent, where J_p is a matrix of ones with order $p \times p$.

6. (10%) Show that

$$A = \begin{pmatrix} 1 + \alpha^2 & -\alpha & 0 & 0 \\ -\alpha & 1 + \alpha^2 & -\alpha & 0 \\ 0 & -\alpha & 1 + \alpha^2 & -\alpha \\ 0 & 0 & -\alpha & 1 \end{pmatrix}$$

is positive definite for $\alpha \in \mathbb{R}$.

7. (15%) Let A be a $p \times p$ matrix with nonnegative eigenvalues. Show that

$$\det(A) \leq \exp[\text{tr}(A - I_p)].$$