國立高雄大學一百學年度研究所碩士班招生考試試題

科目：基礎數學
考試時間：100 分鐘

系所：
統計學研究所(統計組)
本科原始成績：100 分

是否使用計算機：否

1. Evaluate the following integrals:
   (a) (8%) $\int_0^\infty x^5 \exp\{-x^3\}dx$
   (b) (10%) $\int_0^{\pi/2} \frac{\sin x+\sin 2x}{1+\cos^2 x}dx$

2. Evaluate the following limits (if it exists):
   (a) (10%) $\lim_{n\to\infty} (n^{-3}+n^{-2})\sum_{k=1}^{n}(k^2+k)$
   (b) (8%) $\lim_{x\to 0} \frac{\sin 5x}{\sin x}$

3. (12%) A function f is continuous everywhere and satisfies the equation
$$\int_0^x f(t)dt = -\frac{1}{2} + x^2 + x\sin 2x + \frac{1}{2}\cos 2x$$
for all real x. Compute $f(\pi/4)$ and $f'(\pi/4)$.

4. (12%) Given that the derivative $f'(a)$ exists. Derive $\lim_{h\to 0}[f(a+2h)-f(a+h)]/2h$ in term of $f'(a)$

5. (20%) If A and $I+A$ are $k\times k$ nonsingular matrices, show that
   (a) $A^{-1}+I$ is nonsingular
   (b) $(A+I)^{-1}+(A^{-1}+I)^{-1}=I$

6. (20%) Let S be spanned by $x_1^T=[1,1,0,-1]$ and $x_2^T=[0,1,1,1]$; the vector y is defined by $y^T=[1,2,-1,1]$. Find vector z_1 and z_2 such that $y=z_1+z_2$ where z_1 is in S and z_2 is in $S^\perp$.