國立高雄大學一百學年度研究所碩士班招生考試試題

科目：機率論
考試時間：100 分鐘

系所：
統計學研究所(統計組)
本科原始成績：100 分

是否使用計算機：否

1. For each of the following, determine the value of c that makes $f(x)$ a pdf.

   (a) $f(x) = c\sin x,\ 0 < x < \pi/2$ (b) $f(x) = ce^{-|x|},\ -\infty < x < \infty$. (10%)

2. Let X have the pdf

$$f(x) = \frac{4}{\beta^3\sqrt{\pi}} x^2 e^{-x^2/\beta^2},\quad 0 < x < \infty,\quad \beta > 0.$$

   (a) Verify that $f(x)$ is a pdf. (b) Find EX and $VarX$. (10%)

3. Let X be a random variable and let $g(x)$ be a nonnegative function. Then, for any $r > 0$, show that

$$P(g(X) \geq r) \leq \frac{Eg(X)}{r}. \qquad (8\%)$$

4. A particular industrial product is shipped in lots of 20. Testing to determine whether an item is defective is costly, and hence the manufacturer samples his production rather than using a 100% inspection plan. A sampling plan constructed to minimize the number of defectives shipped to customers calls for sampling 5 items from each lot and rejecting the lot if more than 1 defective is observed. (If the lot is rejected, each item in it is later tested.) If a lot contains 4 defectives, what is the probability that it will be rejected? What is the expected number of defectives in the sample of size 5? What is the variance of the number of defectives in the sample of size 5? (10%)

5. Suppose that Y_1 and Y_2 are independent and that both are uniformly distributed on the interval $(0,1)$, and let $U_1 = Y_1 + Y_2$ and $U_2 = Y_1 - Y_2$.

   (a) Show that the joint density of U_1 and U_2 is given by

$$f_{U_1,U_2}(u_1,u_2) = \begin{cases} 1/2, & -u_1 < u_2 < u_1,\ 0 < u_1 < 1 \text{ and} \\ & u_1 - 2 < u_2 < 2 - u_1,\ 1 \leq u_1 < 2 \\ 0, & \text{otherwise.} \end{cases}$$

   (b) Sketch the region where $f_{U_1,U_2}(u_1,u_2) > 0$.

   (c) Show that the marginal density of U_1 is

$$f_{U_1}(u_1)=\begin{cases} u_1, & 0<u_1<1 \\ 2-u_1, & 1\le u_1<2 \\ 0, & \text{otherwise.} \end{cases}$$

(d) Show that the marginal density of U_2 is

$$f_{U_2}(u_2)=\begin{cases} 1+u_2, & -1<u_2<0 \\ 1-u_2, & 0\le u_2<1 \\ 0, & \text{otherwise.} \end{cases}$$

(e) Are U_1 and U_2 independent? Why or why not? (20%)

6. Let X have a Poisson distribution with parameter λ.

(a) Show that the moment-generating function of $Y=(X-\lambda)/\sqrt{\lambda}$ is given by

$$m_Y(t)=\exp(\lambda e^{t/\sqrt{\lambda}}-\sqrt{\lambda}t-\lambda).$$

(b) Use the expansion

$$e^{t/\sqrt{\lambda}}=\sum_{i=0}^{\infty}\frac{[t/\sqrt{\lambda}]^i}{i!}$$

to show that

$$\lim_{\lambda\to\infty} m_Y(t)=e^{t^2/2}.$$

(c) Give a conclusion about the convergence of distribution function of Y. (12%)

7. Suppose that $Y_1,Y_2,\ldots,Y_{40}$ denote a random sample of measurements on the proportion of impurities in iron ore samples. Let each variable Y_i have a probability density function given by

$$f(y)=\begin{cases} 3y^2, & 0\le y\le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

The ore is to be rejected by the potential buyer if $\overline{Y}$ exceeds 0.7. Find $P(\overline{Y}>0.7)$ for the

sample of size 40. (5%)

8. Let the moment-generating function $m(t) = (1/6)e^{t} + (2/6)e^{2t} + 3/6e^{3t}$. Find the following:

   (a) $E(Y)$. (b) $Var(Y)$. (c) the distribution of Y. (10%)

9. Let $X, X_1, X_2 \cdots$ be identically distributed random variables. The joint distribution of (X_n, X) be as followings:

X \ X_n	0	1	
0	0	1/2	1/2
1	1/2	0	1/2
	1/2	1/2	1

   for each n. Do the sequence of random variables $X_1, X_2 \cdots$ converge in probability to X? (5%)

10. A study of the posttreatment behavior of a large number of drug abusers suggests that the likelihood of conviction within a 2-year period after treatment may depend upon the offenders education. The proportions of the total number of cases falling in four education-conviction categories in the following table.

	Status Within Two Years After Treatment		
Education	Convicted	Not Convicted	Totals
10 years or more	0.10	0.30	0.40
9 years or less	0.27	0.33	0.60
Totals	0.37	0.63	1.00

   Suppose that a single offender is selected from the treatment program. Define the events:

   A: The offender has 10 or more years of education.

   B: The offender is convicted within 2 years after completion of treatment.

   Find:

   (a) $P(\overline{A \cup B})$.

   (b) $P(\overline{AB})$.

科目：機率論
考試時間：100 分鐘

系所：
統計學研究所(統計組)
本科原始成績：100 分

是否使用計算機：否

(c) $P(A \mid B)$. (10%)