

國立高雄大學一百學年度研究所碩士班招生考試試題

科目：微積分  
考試時間：100 分鐘

系所：  
統計學研究所(風險管理組)  
本科原始成績：100 分

是否使用計算機：否

1. Evaluate the following integrals:

(a) (10%)  $\int_0^{\infty} x^{20} \exp\{-3x\} dx$

(b) (10%)  $\int_0^{\infty} x^4 \exp\{-x^2\} dx$

(c) (10%)  $\int_0^{\pi/8} (\sin x \cos^3 x)/(2 \cos^2 x - 1) dx$

2. Evaluate the following limits (if it exists):

(a) (10%)  $\lim_{n \rightarrow \infty} (1 + \frac{1}{n^2-1})^{2n^2+1}$

(b) (10%)  $\lim_{n \rightarrow \infty} (x + \exp\{2x\})^{1/x}$

3. Determine whether the following sequences converges or diverges:

(a) (10%)  $(n^2 + 2n - 2)/(3n^2)$

(b) (10%)  $f(n) = n^{2/3} \sin(n!)/(n + 1)$

4. (15%) Let  $f(x) = x - x^2, g(x) = ax$ . Determine  $a$  so that the region above the graph of  $g$  and below the graph of  $f$  has area  $9/2$ .

5. (15%) Show that the equation  $x^2 = x \sin x + \cos x$  has exactly two real roots of  $x$ .