國立高雄大學九十九學年度研究所碩士班招生考試試題

科目：微積分　　系所組別：統計學研究所風險管理組　　是否使用計算機：否

考試時間：100 分鐘　　本科原始成績：100 分

1. (5%) Prove that $2\sqrt{x} > 3 - \frac{1}{x}$ for all $x > 1$.

2. (10%) Find the area of the largest rectangle that can be inscribed in a semicircle of radius 2.

3. (10%) Prove that $|\sin a - \sin b| \leq |a - b|$ for all a and b.

4. (15%) Find the 4th degree Taylor polynomials centered at 0 for the functions $f_1(x) = \cos x$, $f_2(x) = \ln(1 + x)$ and $f_3(x) = \sinh x$, respectively.

5. (30%) Evaluate the following:

   (a) (7%) $\lim_{x \to 5} \frac{x^2}{x - 5} \int_5^x \frac{\tan u - \sin u}{u} du$

   (b) (7%) $\lim_{x \to \infty} \left(1 + \frac{2}{x + 1}\right)^{\ln x}$

   (c) (8%) $\lim_{x \to \infty} (\sqrt{x^2 - 1} - x)$

   (d) (8%) $\lim_{n \to \infty} \frac{1}{n}\left[(\frac{1}{n})^9 + (\frac{2}{n})^9 + \cdots + (\frac{n}{n})^9\right]$

6. (30%) Evaluate the integral:

   (a) (7%) $\int_0^{\sqrt{3}} \frac{x}{x^4 + 9} dx$

   (b) (7%) $\int_0^1 \ln x dx$

   (c) (8%) $\int_0^{\frac{\pi}{2}} e^x \sin x dx$

   (d) (8%) $\int_0^3 \frac{1}{x - 1} dx$