國立高雄大學九十八學年度碩士班招生考試試題

科目：統計學　　　　　　　　　系所：統計學研究所風險管理組　　　　是否使用計算機：否
考試時間：100 分鐘　　　　　　本科原始成績：100 分

1. If $P(A) > 0$, $P(B) > 0$, and $P(A) < P(A \mid B)$, show that $P(B) < P(B \mid A)$. (8%)

2. Find $E[Y(Y-1)]$ for a geometric random variable Y with the following density function

$$p(y) = q^{y-1} p, \quad y = 1,2,3,\cdots, \quad 0 \le p \le 1, \quad q = 1-p,$$

by finding $\dfrac{d^2}{dq^2}\left(\sum_{y=1}^{\infty} q^y\right)$. Use this result to find the variance of Y. (10%)

3. The joint density function of Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} 30 y_1 y_2^2, & y_1 - 1 \le y_2 \le 1 - y_1,\ 0 \le y_1 \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the marginal density of Y_1.
(b) Derive the marginal density of Y_2.
(c) Derive the conditional density of Y_2 given $Y_1 = y_1$.
(d) Find $P(Y_2 > 0 \mid Y_1 = 0.75)$. (10%)

4. Let $X, X_1, X_2 \cdots$ be identically distributed random variables. The joint distribution of (X_n, X) be as followings:

X \ X_n	0	1	
0	0	1/2	1/2
1	1/2	0	1/2
	1/2	1/2	1

for each n. Do the sequence of random variables $X_1, X_2 \cdots$ converge in probability to X? (10%)

5. Let X and Y denote the interest rates that will be paid on one-year certificates of deposit that are issued on the first day of next year (year 1) and the following year (year 2), respectively. The bivariate probability distribution of X and Y follow:

	Year 2 (Y)		
Year 1 (X)	9	10	11
9	0.1	0.1	0
10	0.1	0.3	0.2
11	0	0.1	0.1

Which of the following statement is false?

(a) $P(X < Y) = 0.3$.

(b) The interest rate is expected to be higher in year 2 than in year 1.

(c) There is more uncertainty about the interest rate in year 2 than in year 1.

(d) An investment fund plans to buy $10 million of the one-year CD issued on the first day of year 1 and to reinvest this amount and the earned interest in the CD issued in year 2. Assume that interest on CD is paid at the end of the year they are issued. Then the expected value of this investment, including earned interest, at the end of year 2, is $12.11 million.

(e) Suppose $Z = \max(X, Y)$ than Z must necessarily have a larger expected value than either X or Y. (20%)

6. Let $Y_1, Y_2, \cdots, Y_n$ denote a random sample of size n from a population whose density is given by

$$f(y) = \begin{cases} 3\beta^3 y^{-4}, & \beta \le y \\ 0, & \text{elsewhere} \end{cases}$$

where $\beta > 0$ is unknown. Consider the estimator $\hat{\beta} = \min(Y_1, Y_2, \cdots, Y_n)$.

(a) Derive the bias of the estimator $\hat{\beta}$.

(b) Derive $\mathrm{MSE}(\hat{\beta})$. (10%)

7. Let $Y_1, Y_2, \cdots, Y_n$ be a random sample from a population with density function

$$f(y \mid \theta) = \begin{cases} \dfrac{3y^2}{\theta^3}, & 0 \le y \le \theta \\ 0, & \text{elsewhere} \end{cases}$$

(a) Show that $Y_{(n)} = \max(Y_1, Y_2, \cdots, Y_n)$ is sufficient for θ.

(b) Show that $Y_{(n)}$ has probability density function

$$f_{(n)}(y \mid \theta) = \begin{cases} \dfrac{3ny^{3n-1}}{\theta^{3n}}, & 0 \le y \le \theta \\ 0, & \text{elsewhere.} \end{cases}$$

(c) Find the UMVUE of θ. (10%)

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8. A certain type of electronic component has a lifetime Y (in hours) with probability density function given by

$$f(y \mid \theta) = \begin{cases} \left(\dfrac{1}{\theta^2}\right) y e^{-y/\theta}, & y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

That is, Y has a gamma distribution with parameters $\alpha = 2$ and θ. Let $\hat{\theta}$ denote the maximum likelihood estimator of θ. Suppose that three such components, tested independently, had life times of 120, 130, and 128 h.

(a) Find the maximum likelihood estimate of θ.

(b) Find $E(\hat{\theta})$ and $V(\hat{\theta})$.

(c) Suppose that θ actually equals 130. Given an approximate bound that you might expect for the error of estimation.

(d) What is the MLE for the variance of Y? (12%)

9. Suppose that $Y_1, Y_2, \cdots, Y_n$ denotes a random sample from the probability density function given by

$$f(y \mid \theta_1, \theta_2) = \begin{cases} \left(\dfrac{1}{\theta_1}\right) \exp\left(-\dfrac{y - \theta_2}{\theta_1}\right), & y > \theta_2 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the likelihood ratio test for testing $H_0 : \theta_1 = \theta_{1,0}$ versus $H_a : \theta_1 > \theta_{1,0}$, with θ_2 unknown.

(10%)