國立高雄大學 101 學年度研究所碩士班招生考試試題

科目：統計學
考試時間：100 分鐘

系所：
統計學研究所(風險管理組)
本科原始成績：100 分

是否使用計算機：否

1. A balanced die is tossed six times and the number on the uppermost face is recorded each time. What is the probability that the numbers recorded are 1, 2, 3, 4, 5, and 6 in any order? (5%)

2. Suppose that two balanced dice are tossed repeatedly and the sum of the two uppermost faces is determined on each toss. What is the probability that we obtain a sum of 3 before we obtain a sum of 7? (5%)

3. Find $E[Y(Y-1)]$ for a geometric random variable Y by finding $d^2/dq^2\left(\sum_{y=1}^{\infty} q^y\right)$. Use this result to find the variance of Y. (10%)
   Note. A random variable Y is said to have a geometric probability distribution if and only if
   $$p(y) = q^{y-1}p, \quad y = 1, 2, 3, \cdots, \quad 0 \le p \le 1.$$

4. Find the distributions of the random variables that have each of the following moment-generating functions:
   (a) $m(t) = [(1/3)e^t + (2/3)]^5$.
   (b) $m(t) = e^t/(2-e^t)$.
   (c) $m(t) = \exp(2(e^t-1))$. (12%)

5. The length of time required by students to complete a 1-hour exam is a random variable with a density function given by
   $$f(y) = \begin{cases} cy^2 + y, & 0 \le y \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$
   (a) Find c.
   (b) Find $F(y)$.
   (c) Use $F(y)$ in (b) to find $F(-1)$, $F(0)$, and $F(1)$.
   (d) Find the probability that a randomly selected student will finish in less than half an hour.
   (e) Give that a particular student needs at least 15 minutes to complete the exam, find the probability that she will require at least 30 minutes to finish. (15%)

6. Find $P(|Y-\mu| \le 2\sigma)$ for the continuous uniform random variable. Compare with the corresponding probabilistic statements given by Tchebysheff's theorem and the empirical rule. (10%)
   Note. If $\theta_1 < \theta_2$, a random variable Y is said to have a continuous uniform probability distribution on the interval (θ_1, θ_2) if and only if the density function of Y is

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$$f(y) = \begin{cases} \dfrac{1}{\theta_2 - \theta_1}, & \theta_1 \le y \le \theta_2 \\ 0, & \text{elsewhere.} \end{cases}$$

7. A learning experiment requires a rat to run a maze (a network of pathways) until it locates one of three possible exits. Exit 1 presents a reward of food, but exits 2 and 3 do not. (If the rat eventually selects exit 1 almost every time, learning may have taken place.) Let Y_i denote the number of times exit i is chosen in successive runnings. For the following, assume that the rat chooses an exit at random on each run.
   (a) Find the probability that $n = 6$ runs result in $Y_1 = 3, Y_2 = 1$, and $Y_3 = 2$.
   (b) For general n, find $E(Y_1)$ and $Var(Y_1)$.
   (c) Find $Cov(Y_2, Y_3)$ for general n.
   (d) To check for the rat's preference between exits 2 and 3, we may look at $Y_2 - Y_3$. Find $E(Y_2 - Y_3)$ and $Var(Y_2 - Y_3)$ for general n. (20%)

8. Let Y_1 and Y_2 be independent random variables, both uniformly distributed on (0, 1). Find the probability density function for $U = Y_1 Y_2$. (8%)

9. Suppose that Y_1 and Y_2 are independent exponentially distributed random variables, both with mean β, and define $U_1 = Y_1 + Y_2$ and $U_2 = Y_1 / Y_2$.
   (a) Show that the joint density of (U_1, U_2) is

$$f(u_1, u_2) = \begin{cases} \dfrac{1}{\beta^2} u_1 \exp\left(-\dfrac{u_1}{\beta}\right) \dfrac{1}{(1+u_2)^2}, & 0 < u_1, 0 < u_2 \\ 0, & \text{otherwise.} \end{cases}$$

   (b) Are U_1 and U_2 independent? Why? (10%)

10. Let $Y_1, Y_2, \cdots, Y_n$ be independent, exponentially distributed random variables with mean β. Give the joint density function for $Y_{(j)}$, the jth-order statistic, and $Y_{(k)}$, the k-th order statistic, where j and k are integers $1 \le j < k \le n$. (5%)