Singular Value Decomposition in High Dimensionality Reduction and Image Compression

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Abstract

The techniques of high dimensionality reduction are important tools in machine learning and data science. The method of singular value decomposition (SVD) is a popular method in dimensionality reduction and image compression. However, it suffers from heavily computational overhead in practice, especially for images with high-resolution. In order to achieve the efficiency and the accuracy, we propose a refinement of approximate invariant supspaces of matrices (REIS) algorithm based on SVD. The theoretical contribution of our paper is threefold. Firstly, we describe the properties of the SVD of the matrices and discuss how to apply SVD to do image compression. Secondly, we introduce the method of REIS based on SVD for image compression in the high-resolution images. The core of REIS is adapted to large and real matrices in $\mathbb{R}^{n\times n}$, through some nonsymmetric algebraic Riccati equations or their associated Sylvester equations via Newton's method. Thirdly, some measurement tools are provided such as compression ratio, mean square error, peak signal to noise ratio and structural similarity index to compare the performance of the compression factors and the quality of the compressed images. Numerical examples for testing some real world image sets are presented to illustrate the feasibility of our proposed algorithm.

Keywords: Singular value decomposition, high dimensionality reduction, high-resolution image compression, refinement of approximate invariant subspace, nonsymmetric algebraic Riccati equation, Newton's method