

# On Counterfactual Hazard

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## 摘要

Hazard plays a critical role in survival analyses ranging from model specification, statistical inference to application. However, limitations of its causal interpretation have also been well recognized. We attempt to tackle the challenging issue by proposing three definitions of counterfactual hazard using conditioning counterfactual survival time (cCT), conditioning counterfactual process (cCP) and interventional counterfactual process (iCP). In the average causal effect setting, the identification formulas of the three definitions coincide under their respective assumptions. We further extend the three counterfactual hazards to a mediation setting with a mediator measured during the follow-up where the mediator has two components: time-to-measurement and the mediator value if measured. The cCT may have difficulties in identification; the cCP suffers from the built-in selection bias due to its definition as a conditional probability. In contrast, the iCP can identify the effect mediated by the status regarding the measurement of the mediator, the effect mediated by the measured value of the mediator, and the effect not mediated by the mediator (either its measurement or actual value). Under the iCP, we propose nonparametric and semiparametric estimators and establish their uniform consistency and weak convergence. Numerical simulation is conducted to assess its finite sample performance. We apply our proposed methods to two studies, investigating the effect of hepatitis B on mortality mediated by the incidence of liver cancer, and the effect of hepatitis C on time-to-liver cancer mediated by time-to-measurement and the actual value of hepatitis B viral load.